

TABLE A3. Expansion Coefficients A_n^N with Standard Errors of W^2 according to (A1) for Shear and Quasi-Shear Modes

Coefficient	Mode No.	\vec{N}	\vec{U}	Sample	$N = 2$ and $n = 2, 2$	$N = 3$ and $n = 2, 2$	$N = 3$ and $n = 3, 3$
					$10^{-8} \text{cm}^2 \text{sec}^{-2} \text{kb}^{-2}$	$10^{-8} \text{cm}^2 \text{sec}^{-2} \text{kb}^{-2}$	$10^{-8} \text{cm}^2 \text{sec}^{-2} \text{kb}^{-3}$
c_{44}	1	[010]	[001]	1	-447 ± 27	-685 ± 148	16 ± 10
	2	[010]	[001]	4	-501 ± 35	-605 ± 205	7 ± 13
	3	[001]	[010]	1*	-392 ± 36	-486 ± 100	10 ± 8
	4	[001]	[010]	1	-361 ± 24	-711 ± 100	22 ± 6
	Average				-425 ± 31	-622 ± 51	14 ± 3
c_{55}	5	[100]	[001]	1	-917 ± 24	-1127 ± 132	13 ± 8
	6	[100]	[001]	3	-806 ± 33	-1306 ± 134	32 ± 8
	7	[001]	[100]	1	-881 ± 45	-1584 ± 192	46 ± 12
	Average				-868 ± 33	-1339 ± 96	31 ± 10
c_{66}	8	[100]	[010]	1	-253 ± 22	30 ± 119	-19 ± 8
	9	[100]	[010]	3*	-200 ± 35	298 ± 155	-32 ± 10
	10	[010]	[100]	1	-257 ± 56	-723 ± 309	30 ± 20
	11	[010]	[100]	4	-237 ± 28	10 ± 151	-16 ± 10
	Average				-237 ± 13	-385 ± 459	-9 ± 50
c_{12}	12	[$l\bar{m}0$]	[$m\bar{l}0$]	2*	-459 ± 45	-714 ± 256	17 ± 17
	13	[$l\bar{m}0$]	[$m\bar{l}0$]	2	-448 ± 76	-1037 ± 419	40 ± 21
	Average				-454 ± 35	-876 ± 115	29 ± 8
c_{13}	14	[$l0n$]	[$n0\bar{l}$]	4*	-632 ± 71	1 ± 374	-44 ± 25
	15	[$l0n$]	[$n0\bar{l}$]	4*	-659 ± 83	-806 ± 491	10 ± 33
	Average				-646 ± 10	-403 ± 285	-17 ± 19
c_{23}	16	[$0m\bar{n}$]	[$0n\bar{m}$]	3*	-375 ± 24	-498 ± 139	8 ± 9
	17	[$0m\bar{n}$]	[$0n\bar{m}$]	3	-349 ± 33	395 ± 201	2 ± 3
	Average				-362 ± 9	-52 ± 315	5 ± 2

*Run made with Arenberg PSP AFC ultrasonic equipment. All other data were taken with MRL PSP AFC equipment.

those for $N = 2$ and $N = 3$, and it is therefore reasonable to expect that an increase of 50% represents an upper bound for the truncation error. To eliminate or reduce the truncation error for A_2^N , all measurements would have to be extended to substantially higher pressures and the data fitted to a polynomial of degree N greater than 3 or 4, such that this fit would still be statistically significant and A_2^N would become independent of N within its standard deviation. This task remains for the future.

Acknowledgments. We would like to thank Drs. Z. P. Chang and H. E. Shull and Mr. P. L. Carcia for technical advice and Mrs. J. Schiff for the computer work in connection with the data analysis. Our thanks are extended to Dr. E. K. Graham for bringing to our attention P. W. Bridgman's compression data on hypersthene. Further, Mr. H. H. Demarest's suggestion of equation 8 for the error calculation and a fruitful discussion with Dr. B. L. Joiner and Miss K. McKim of the Pennsylvania State University Statistical Consulting Service on nonlinear regression analysis and on the applicability of (8) are gratefully acknowledged.

This work was supported by the National Science Foundation under grant GA 3985.

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